Graph track description

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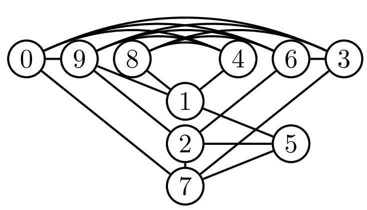
1 Results

The sizes of the reconfiguration sequences in each track are given in the table below.

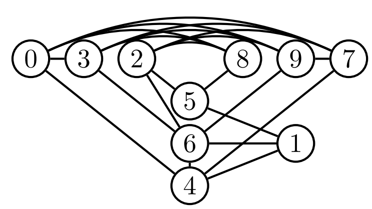
Size n of the graph	Size k of the IS	Length of the sequence
10	4	10
50	15	3410
100	30	3495250

2 Construction for $n = 10$

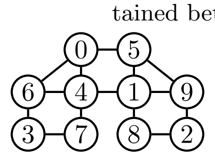
By bruteforcing all graphs on $n = 10$ vertices, we obtain that three graphs maximize the diameter of the reconfiguration graph. In each case, the reconfiguration graph is a path on 11 vertices. The three graphs are depicted below:



(a) Graph G_1, maximum distance obtained between 167 and 789.



(b) Graph G_2, maximum distance obtained between 012 and 056.



(c) Graph G_3, maximum distance obtained between 0123 and 2345.

3 Construction for $n = 50$ and $n = 100$

The construction for higher number of vertices is based on the graph G_1 depicted before. We draw below the reconfiguration graph of 3-IS in G_1.

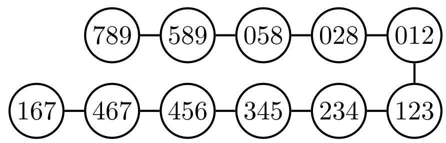


Figure 2: The reconfiguration graph of 3-IS in G_1.

Our construction relies on the following operation. Starting from an instance (G, α, β), we construct an instance (G', α', β') as follows:

- Add 10 new vertices to G inducing G_1. We assume that these vertices are labeled with $0, \ldots, 9$ according to Figure 1a.
- For every vertex u of G not in α, we add an edge in G' between u and the vertex 1 of G_1. We also add, for every vertex $v \in G$ not in β an edge in G' between v and 5.
- We set $\alpha' = \alpha \cup \{7, 8, 9\}$ and $\beta' = \alpha \cup \{7, 8, 9\}$.

We claim that this construction satisfies the following.

- $|V(G')| = |V(G)| + 10$;
- $|\alpha'| = |\alpha| + 3$ and α' is a maximum independent set in G';
- The connected component of the reconfiguration graph of G containing α' and β' is a path whose endpoints are α' and β'.
- $d' = 4d + 10$ where d (resp. d') is the distance between α and β (resp. α' and β') in the reconfiguration graph of G (resp. G').

The first point is straightforward. The second point also holds. Indeed, α' is a maximum independent set since α is a maximum independent set of G and G_1 has independence number 3. Since α is a maximum independent set of G, there must always be three tokens in G_1, and these tokens can only move following the reconfiguration sequence of G_1. Therefore, at each step, one can either move a token in G (following the reconfiguration graph of G) or in G_1.

Now observe that tokens in G_1 cannot move unless the tokens in G induce α or β. Conversely, tokens in G cannot move unless the tokens in G_1 induce an IS not containing 1 nor 5. This ensures that the reconfiguration graph of G' is a path with endpoints α' and β'.

One can finally check that the length d' of this path is $4d+10$. In order to put tokens in position 028 in G_1, one needs to put a token in vertex 5 (see Figure 2), which implies to put all tokens of G in β. This requires d steps, hence in $d + 3$

steps, we manage to obtain the independent set $\beta \cup \{0, 2, 8\}$. Now, similarly, to put the tokens on 234, we need to put tokens in 1, which requires to put back the tokens of G on α. By iterating again this argument twice, we finally obtain a transformation where the total number of token slides in G is $4d$ times, while the total number of token moves in G_1 is 10, which completes the proof.

In particular, applying several times this construction starting from the instance $(G_1, \{7, 8, 9\}, \{1, 6, 7\})$, we can construct instances (G, α, β) where G has n vertices, α, β are independent sets of size $3n/10$ at distance $\frac{10}{3}(4^{n/10} - 1)$ in the reconfiguration graph (note that their component is a path).

Note that the same construction works when replacing G_1 by the complement of a path on 5 vertices, linking the middle vertex to β. However, in that case we get $|V(G')| = |V(G)| + 5$ and $d' = 2d + 3$, hence we get graphs on n vertices with a reconfiguration sequence of size $3 \times (2^{n/5} - 1)$, which is slightly worse. This also means that this construction can be improved with a better choice of G_1.