

# The Documentation of Graph Track in Core Challenge

Takashima Yuya<sup>1</sup> and Yamaoka Chuta<sup>1</sup>

<sup>1</sup>Minato laboratory of Kyoto University

31 March 2022

## 1 Graph Structure

Consider a piece consisting of 5 vertices and 2 tokens (Fig. 1). This piece can switch between 0-state and 1-state via transition-state as shown in Figure 2.

Here, if vertex 1 is blocked by an external token in the 0-state or 1-state state, this piece will not be able to switch states (Fig. 3). By preparing piece-1 and piece-2 and connecting them with edges as shown in Figure 4, it is possible to make the possibility of transition of piece-2 depend on piece-1.

Prepare  $n$  pieces piece-1, piece-2 ... piece- $n$ . For  $i(2 \leq i \leq n)$ , stretch the edges so that piece- $i$  is transitive only when piece- $i - 1$  is 1-state and piece- $j(1 \leq j \leq i - 2)$  is 0-state. The initial token placement state is such that each piece is 0-state. The target token placement state is set so that only piece- $n$  is 1-state and all other pieces are 0-state.

Suppose there are  $m$  pieces, and the state of each piece is represented by an 01-string of length  $m$ . In the transition from the initial state to the target state, this 01-string changes by one character, and all  $2^m$  possible 01-strings are transitioned as in the Gray code.

## 2 Reconfiguration Sequence Length

Each piece requires 3 transitions to change its state. There are  $|V|/5$  pieces in the graph, therefore the reconfiguration sequence length is  $3 \times (2^{|V|/5} - 1)$ .

10 vertices: 2 pieces, so the length is  $3 \times (2^2 - 1) = 9$ .

50 vertices: 10 pieces, so the length is  $3 \times (2^{10} - 1) = 3069$ .

100 vertices: 20 pieces, so the length is  $3 \times (2^{20} - 1) = 3145725$ .

Fig.1 A piece consisting of 5 vertices and 2 tokens

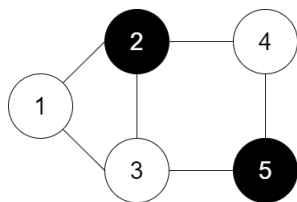


Fig.2 Transition of a piece

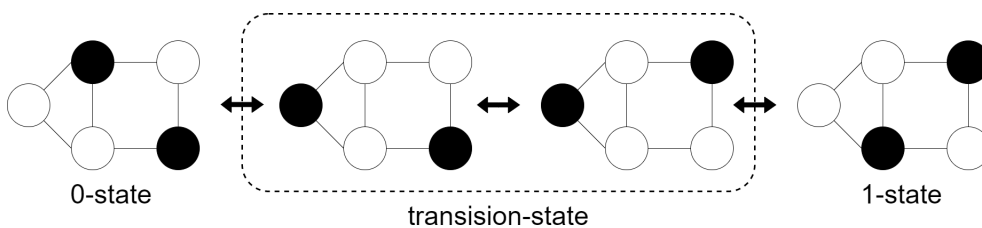


Fig.3 Blocking transitions by external tokens

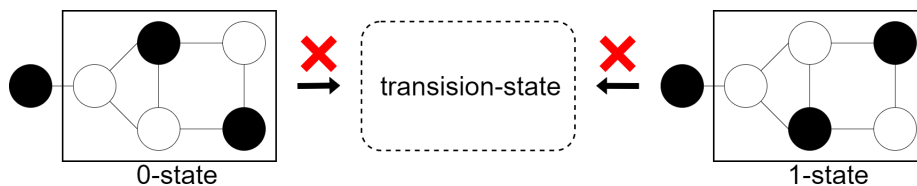


Fig.4 Blocking transitions depending on the state of the external piece

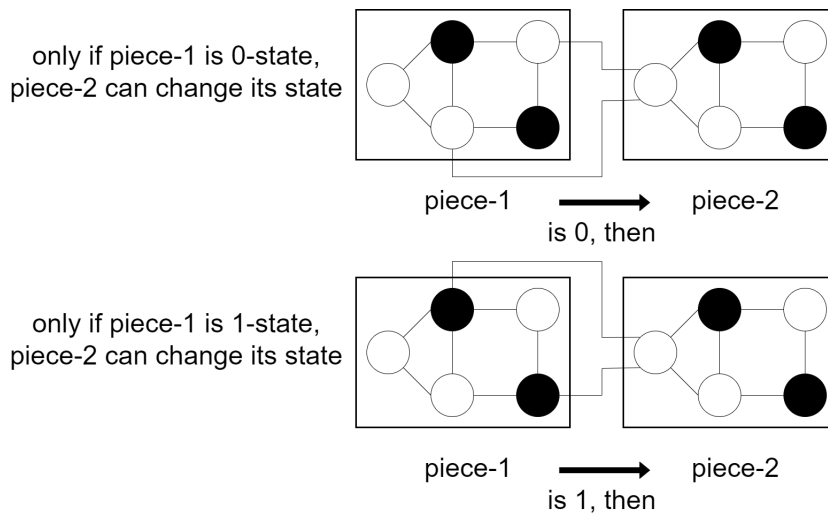


Fig.5 Connection of each piece

